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CONSTRAINED OPTIMIZATION OF HOLE SHAPES IN PLATE STRUCTURES. (U)

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CONSTRAINED OPTIMIZATION OF HOLE SHAPES IN PLATE STRUCTURES

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S. K. Dhir

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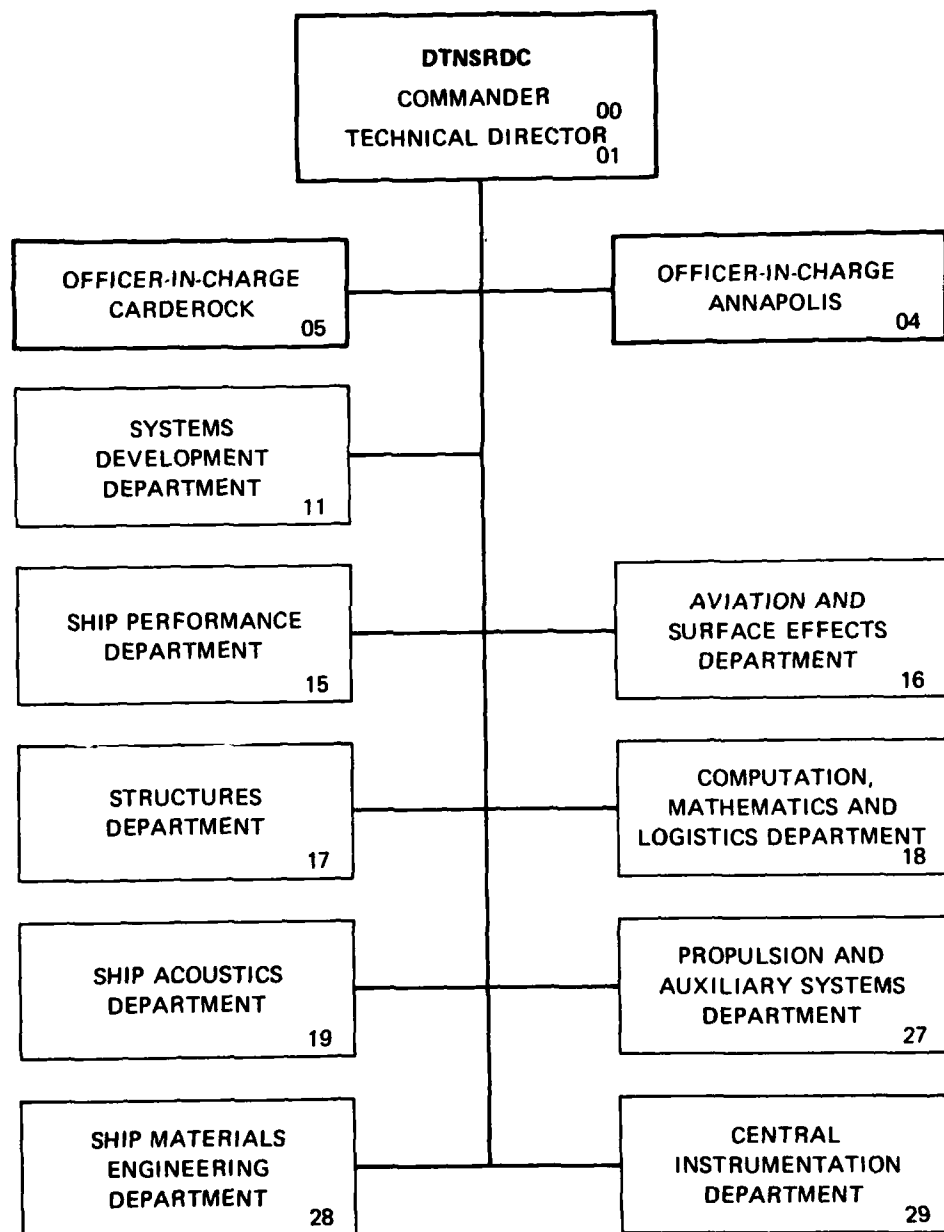
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expressions are then squared and integrated around the hole boundary to obtain the stress integral. The squaring prevents any cancellation of compressive and tensile stresses during the integration process and thus produces a more meaningful integral.

A sample problem is worked out in detail to demonstrate this procedure. The optimum shape of a square-like (double barrel shape) hole with rounded corners was determined by restricting the mapping function to only one specific unknown coefficient. Modification of this shape by introduction of an additional term is also discussed. Numerical stress concentration values for this case are compared with those from other sources.

Most of the algebraic and numerical calculations presented in this paper were performed using MACSYMA, a symbolic manipulation package developed at MIT and in regular use at DTNSRDC.

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## ABSTRACT

An analytical/numerical procedure has been developed which yields the optimum geometry for a constrained hole shape in a large plate under prescribed boundary stresses at infinity. The optimality criterion is based on the minimization of a certain stress integral taken around the hole boundary. Muskhelishvili's method is used to first obtain the symbolic stress expressions for a given mapping function with unknown coefficients. These stress expressions are then squared and integrated around the hole boundary to obtain the stress integral. The squaring prevents any cancellation of compressive and tensile stresses during the integration process and thus produces a more meaningful integral.

A sample problem is worked out in detail to demonstrate this procedure. The optimum shape of a square-like (double barrel shape) hole with rounded corners was determined by restricting the mapping function to only one specific unknown coefficient. Modification of this shape by introduction of an additional term is also discussed. Numerical stress concentration values for this case are compared with those from other sources.

Most of the algebraic and numerical calculations presented in this paper were performed using MACSYMA, a symbolic manipulation package developed at MIT and in regular use at DTNSRDC.

## ADMINISTRATIVE INFORMATION

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## INTRODUCTION

One of the most important problems in the design of plate structures is to determine and minimize the stress concentration due to the inevitable presence of holes and other discontinuities. The literature contains a great deal of information on this age-old problem; however, most of this information is concerned with direct determination of

stresses for given geometry and load conditions. Some recent exceptions are the experimental results reported by Durelli and Rajaiah,<sup>1\*</sup> the harmonic hole concept discussed by Bjorkman and Richards,<sup>2,3</sup> and the optimization approach developed by Schnack.<sup>4</sup> The experimental results of Durelli and Rajaiah were obtained by step-by-step machining of hole boundaries in a photoelastic model until the stress (related directly to the order of isochromatic fringe for load free boundaries) was observed to be approximately constant in both the tensile and compressive boundary regions. This technique is effective, but it does call for a series of experiments for each unique situation. The harmonic hole concept of Bjorkman and Richard conserves the first stress invariant; however, it appears to have limited application as an optimization procedure because it results in somewhat unpractical hole shapes. Schnack's procedure, an interesting finite element approach based on the fade-away law developed by Neuber,<sup>5</sup> is in a way the numerical equivalent of Durelli and Rajaiah's experimental method.

The present paper is a step toward practical design optimization, in that the technique permits the designer to have some control over the general shape of the hole. The problem can thus be formulated: Determine the geometry of a given type of hole (e.g., square-like) which, when placed in a uniformly uni- or biaxially loaded large plate, produces boundary stresses which minimize a certain stress integral. Such an optimum hole appears to lead, in most practical cases, to a very desirable overall stress field.

The approach involves first determining the symbolic boundary stress expression for a desired mapping function in a fairly general form following the well established Muskhelishvili method. This step may require considerable algebra depending on the mapping function. In the present case most of these algebraic operations and the following steps were accomplished by MACSYMA, a symbolic manipulation package developed at MIT. The next step is to square this expression and integrate it around the hole so that the tensile and compressive stresses do not annihilate each other during integration. The last step is to minimize this expression with respect to the mapping function coefficients. These

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\* A complete listing of references is given on page 13.



extreme values of coefficients contain the necessary information to define the geometry of the desired opening.

### STRESS DETERMINATION

Let the mapping function be represented by

$$z(\zeta) = \zeta + \frac{m_1}{\zeta} + \frac{m_3}{\zeta^3} + \frac{m_5}{\zeta^5} + \frac{m_7}{\zeta^7} \quad (1)$$

a function which can map a unit circle in  $\zeta$ -plane ( $\zeta = e^{i\theta}$ ) into a hole with at least two areas of symmetry in the  $z$ -plane ( $z = x + iy$ ). The coefficients  $m_n$  ( $n=1, 3, 5, 7$ ) are assumed to be real and have the usual limitations on them which assure that  $z'(\zeta) \neq 0$  for  $\zeta \geq 1$  (prime indicates differentiation with respect to the argument). The stress field can be represented by

$$\sigma_\alpha + \sigma_\beta = 2 \left[ \frac{\phi'(\zeta)}{z'(\zeta)} + \frac{\overline{\phi'(\zeta)}}{\overline{z'(\zeta)}} \right] \quad (2)$$

and

$$\sigma_\beta - \sigma_\alpha + 2i\tau_{\alpha\beta} = \frac{2\zeta^2}{\rho^2 z'(\zeta)} [\overline{z(\zeta)} \phi'(\zeta) + \psi'(\zeta)] \quad (3)$$

where

$$\phi(\zeta) = \frac{\phi'(\zeta)}{z'(\zeta)}; \sigma_\alpha, \sigma_\beta \text{ and } \tau_{\alpha\beta} \text{ are the usual plane stresses;}$$

$\rho$  is the absolute value of  $\zeta$ ; and bar (-) over a function represents its complex conjugate. The functions  $\phi(\zeta)$  and  $\psi(\zeta)$  are known to have the form

$$\phi(\zeta) = S\zeta + \sum_{n=1}^{\infty} \frac{a_n}{\zeta^n} \quad (4)$$

$$\psi(\zeta) = D\zeta + \sum_{n=0}^{\infty} \frac{b_n}{\zeta^n} \quad (5)$$

where  $a_n$  and  $b_n$  are constants to be determined and

$$\left. \begin{aligned} S &= \frac{p+q}{4} \\ D &= \frac{p-q}{2} e^{-2i\theta} \end{aligned} \right\} \quad (6)$$

where  $p$  and  $q$  represent the uniform stresses at infinity at an angle  $\theta$  to the  $x$ - and  $y$ -axes, respectively. The  $a_n$  were determined using Muskhelishvili's procedure for a load-free hole boundary and  $\theta=0$ , and are given in the appendix for completeness sake.

#### OPTIMIZATION PROCEDURE

Since the hole boundary ( $\rho=1$ ) is assumed to be load-free ( $\sigma_\alpha = \tau_{\alpha\beta} = 0$ ), the stresses around its boundary can be described by equation (2) which requires the determination of  $\phi(\zeta)$ , i.e.,  $a_n$  only. Now the mapping function  $z(\zeta)$ , given by Equation (1), can be redefined in terms of  $Q_n$ , such that

$$\left. \begin{aligned} m_1 &= \sum_{i=1}^4 Q_i \\ m_3 &= -\frac{1}{3} \sum_{i=1}^3 \sum_{j=i+1}^4 Q_i Q_j \\ m_5 &= \frac{1}{5} \sum_{i=1}^2 \sum_{j=i+1}^3 \sum_{k=j+1}^4 Q_i Q_j Q_k \\ m_7 &= -\frac{Q_1 Q_2 Q_3 Q_4}{7} \end{aligned} \right\} \quad (7)$$

These substitutions, which can be reduced to a cubic equation for determining  $Q_n$ , permit  $z'(\zeta)$  to be given by

$$z'(\zeta) = \prod_{n=1}^4 \left( 1 - \frac{Q_n}{\zeta^2} \right) \quad (8)$$

Clearly  $|Q_n| < 1$ , and the stresses then can be represented at the hole boundary by

$$\sigma_{\beta} = 4 \operatorname{Re} \frac{S - \sum_{n=1}^4 \frac{(2n-1)a_{2n-1}}{\sigma^{2n}}}{\prod_{n=1}^4 \left(1 - \frac{Q_n}{\sigma^2}\right)} \quad (9)$$

which can conveniently be simplified to

$$\sigma_{\beta} = 4 \operatorname{Re} \left[ -S + \sum_{n=1}^4 \frac{P_n}{\left(1 - \frac{Q_n}{\sigma^2}\right)} \right] \quad (10)$$

where use has been made of the last of Equations (7) as well as that of the value of  $a_7$  from the appendix. The  $P_n$  are defined as follows:

$$P_1 = - \frac{Q_1^4 S - Q_1^3 a_1 - 3Q_1^2 a_3 - 5Q_1 a_5 - 7a_7}{Q_1(Q_2 - Q_1)(Q_3 - Q_1)(Q_4 - Q_1)}$$

$P_2$ ,  $P_3$ , and  $P_4$  can be obtained by cyclically rotating  $Q_n$  in the above expression. Equation (10) reduces to

$$\sigma_{\beta} = 4 \left[ -S + \sum_{n=1}^{\infty} (P_1 Q_1^{n-1} + P_2 Q_2^{n-1} + P_3 Q_3^{n-1} + P_4 Q_4^{n-1}) \cos 2(n-1)\beta \right] \quad (11)$$

Although the constants  $P_n$  and  $Q_n$  can be individually complex, the expression to be summed in Equation (11) will always be real for symmetrically located holes and  $\theta=0$ . Parseval's theorem can now be used to show that

$$I = \frac{1}{16\pi} \int_0^{2\pi} \sigma_{\beta}^2 d\beta = 2 \left( S - \sum_{n=1}^4 P_n \right)^2 - \sum_{n=1}^4 \left( \sum_{m=1}^4 P_m \right)^2 + \sum_{m=1}^4 \sum_{n=1}^4 \frac{P_m P_n}{1 - Q_m Q_n} \quad (12)$$

where  $I$  is some form of a measure of the strain energy around the hole. It should be noted that the integration in Equation (12) is performed around  $\beta$  and not around the polar angle  $\theta$ . Equation (12) can be differentiated with respect to  $Q_n$  and equated to zero to minimize  $I$ . By introducing, for example, some form of Lagrange's multipliers, it is possible to introduce additional constraints.

Whenever more than one constant,  $m_n$  or  $Q_n$ , is involved in the mapping function, the expression for  $I$ , Equation (12), explicit in  $Q_n$  only, becomes so complex that its derivatives with respect to  $Q_n$  produce unwieldy expressions. For this reason, it may be desirable in such cases to simply compute the numerical values of  $I$  over a range of parameters and thus determine the minimum point.

Equations (7) thru (12) have been developed for the mapping function given by Equation (1). Therefore, these expressions should be directly reducible to accommodate other mapping functions with any of the  $m_n$  equal to zero. While this is generally true, some expressions, such as the cyclic definitions of  $P_n$ , appear to have poles in some situations. However, these are not real poles and are easily removable as the reader can verify. Special attention should be paid when programming these general expressions for a computer.

#### EXAMPLE I

Let the mapping function be given by

$$z = \zeta + \frac{m_3}{\zeta^3}, \quad |m_3| < \frac{1}{3}; \quad (13)$$

which for non-zero values of  $m_3$  can map a unit circle into a square with rounded corners. The actual geometry of the opening (e.g., rounded corners) depends on the value of  $m_3$ . Equations (7) and (10) can be used to obtain

$$\left. \begin{aligned} Q_1 &= -Q_2 = \sqrt{3m_3} \\ Q_3 &= Q_4 = 0 \\ P_1 &= S + \frac{3D}{2(3-Q_1^2)Q_1} \\ P_2 &= S + \frac{3D}{2(3-Q_2^2)Q_2} \\ P_3 &= P_4 = 0 \end{aligned} \right\} \quad (14)$$

The expressions for  $Q_n$  and  $P_n$  given in Equations (14) can be substituted in Equation (11) and reduced to

$$\sigma_\beta = 4 \frac{(1-m_3)(1-9m_3^2)S + (1-3m_3)D \cos 2\beta}{(1-m_3)(1+9m_3^2 - 6m_3 \cos 4\beta)} \quad (15)$$

These values of  $Q_n$  and  $P_n$ , when substituted in Equation (12), yield the integral I as

$$I = -2S^2 + \frac{4S^2}{1-9m_3^2} + \frac{D^2}{(1-m_3)^2(1-9m_3^2)} \quad (16)$$

The first derivative of this integral with respect to  $m_3$  yields the following condition for extremum values of I:

$$36 S^2 m_3^4 - 108 S^2 m_3^3 + 18(D^2 + 6S^2)m_3^2 - 9(D^2 + 4S^2)m_3 - D^2 = 0 \quad (17)$$

The symbolic solution of Equation (17) was obtained as a function of S and D, but it is not included here because of space limitations. Four specific cases were considered:

- (i) isotropic loading,  $p = q = 1$ ;
- (ii) uniaxial loading,  $p = 1, q = 0$ ;
- (iii) biaxial loading,  $p = 2, q = 1$ ; and
- (iv) shear loading,  $p = 1, q = -1$ .

The values of  $m_3$  which were found, using the solution of Equation (17), to be within the bound of Equation (13) (i.e.,  $|m_3| < \frac{1}{3}$  for the four cases) are, respectively:

- (i)  $m_3 = 0$ ;
- (ii)  $m_3 = -.049297$  ( $\approx -0.05$ );
- (iii)  $m_3 = -0.0107713$  ( $\approx -0.01$ ); and
- (iv)  $m_3 = -0.0936$  ( $\approx -0.09$ ).

#### Case (i)

The first case is obvious, since  $m_3 = 0$  represents a circular opening which is optimum in an isotropic stress field characterized by  $p = q = 1$ . The value of I is 0.5.

#### Case (ii)

The second case ( $m_3 = -0.05, p = 1, q = 0$ ) is worthy of comment.

The value of  $m_3$  used by Savin<sup>6</sup> for a square hole with rounded corners and somewhat concave sides is  $-1/6$ . A modified version with fairly straight sides and rounded corners can be obtained by taking  $m_3 = -1/8$ . For comparison, the values of the integral,  $I$ , for this case were found to be 0.3627, 0.3958, and 0.4532 for  $m_3 = -0.05$ ,  $-1/8$ , and  $-1/6$ , respectively. Figure 1 compares the shape of this optimized hole ( $m_3 = -0.05$ ) with the shapes of holes obtained by setting  $m_3 = -1/8$  and  $-1/6$ . Figure 2 compares the boundary stress distribution for these holes plotted as a function of  $\beta$ . The highest stress is smallest in the case of optimized shape. The boundary stresses for the optimized shape discussed by Durelli and Rajaiah<sup>1</sup> are also included; however, a complete distribution as a function of  $\beta$  could not be plotted as the relationship between  $\beta$  and the polar angle for this shape is not known.

Case iii:

The third case ( $m_3 = -0.01$ ,  $p = 2$ ,  $q = 1$ ) represents only a slight perturbation on a circular hole; nevertheless, for this stress field, it is the optimum shape for a hole whose boundary is constrained by a function given by Equation (13). The value of the integral for this case is 1.3723.

The calculations which have been performed for Equation (13) were repeated for the elliptical shape, i.e.,

$$z = \zeta + \frac{m_1}{\zeta}; \quad |m_1| < 1 \quad (18)$$

The value of the integral in this case was found to be

$$I = 2S^2 + \frac{(2Sm_1 + D^2)}{1-m_1^2} \quad (19)$$

which for a minimum value requires

$$m_1 = -\frac{D}{2S} \quad (20)$$

It is interesting to note that this value of  $m_1$  coincides with that of Bjorkman and Richards<sup>2</sup> for their harmonic hole. The value of the integral in this case is simply  $2S^2$ , i.e., 1.125 when  $p = 2$  and  $q = 1$  and  $m_1 = 1/3$ .

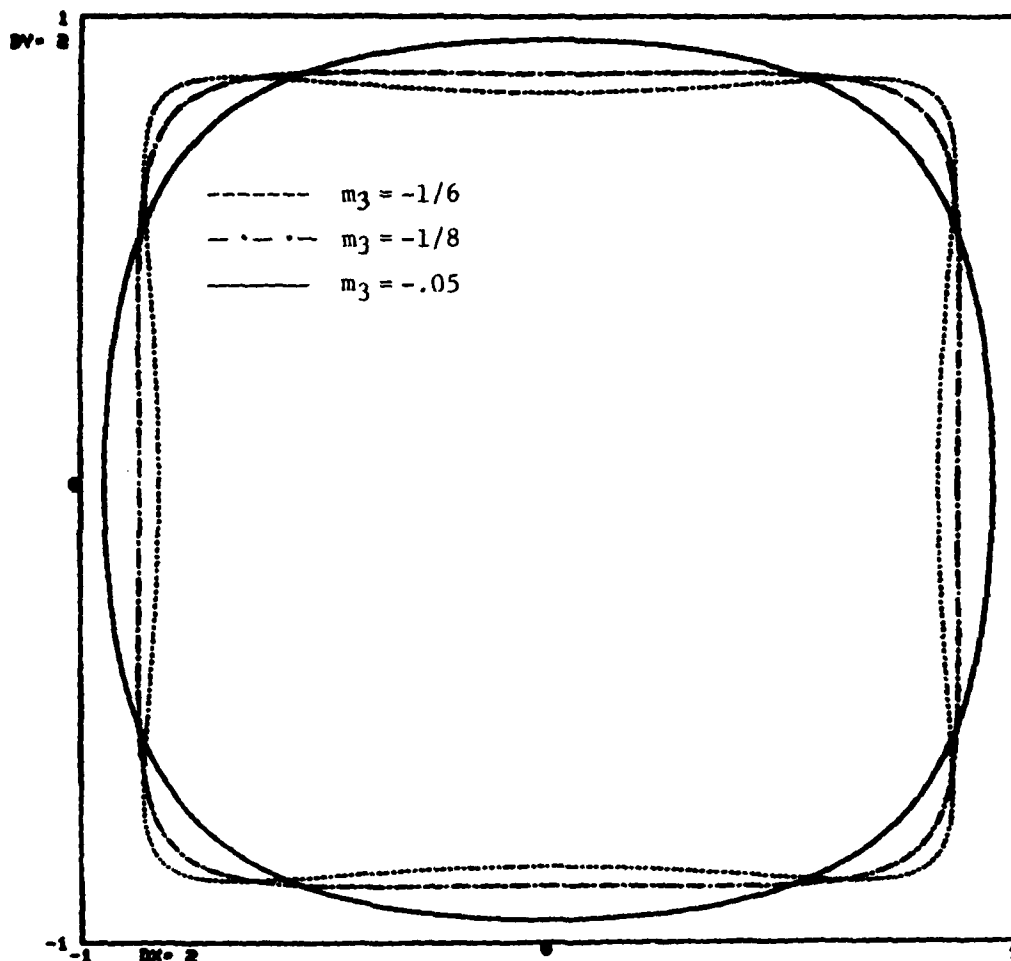


Figure 1 - Comparison of Hole Shapes

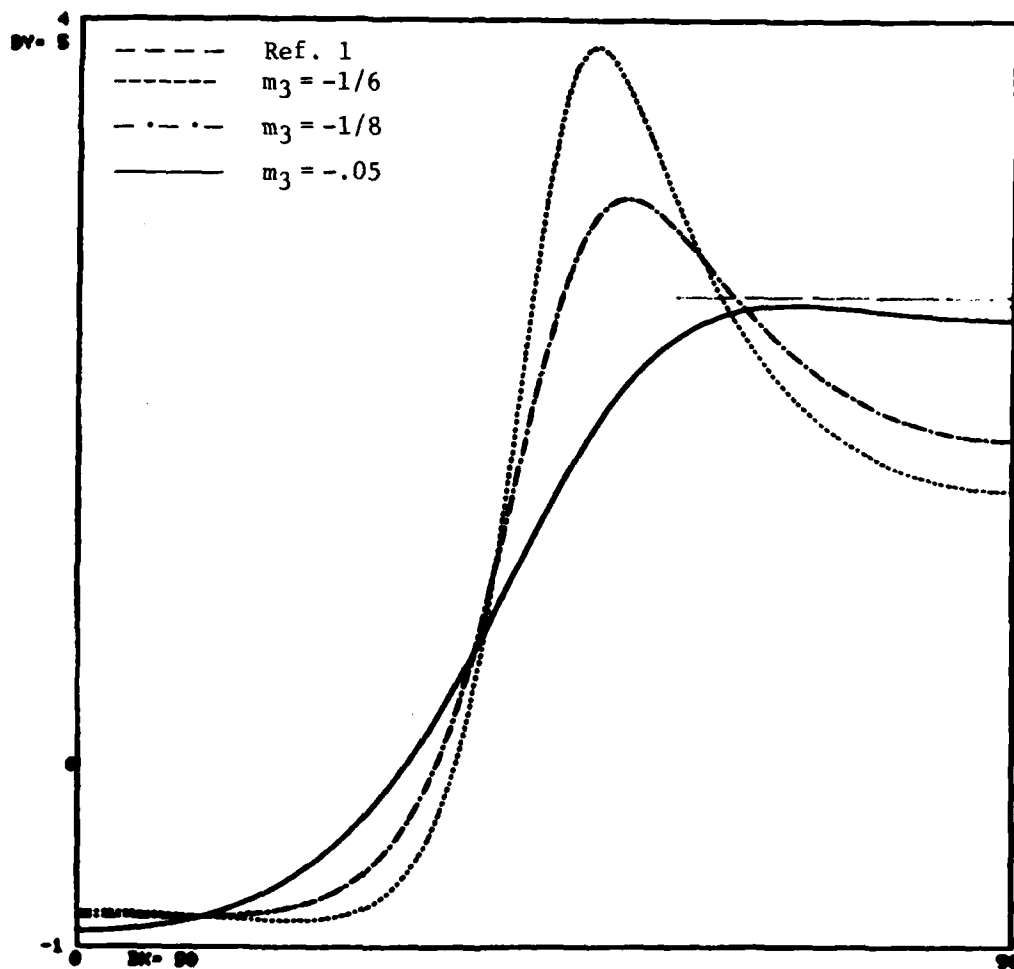


Figure 2 - Comparison of Boundary Stresses



A comparison of this elliptical case with case (iii) of the square-like hole (Equation (13)) shows that, of all the shapes considered, the elliptical hole produces the smallest stress concentrations under a biaxial stress field. On the basis of the work of Bjorkman and Richards,<sup>2</sup> this conclusion can be generalized to include any hole shape. However, if the hole shape is constrained to a shape given by Equation (13), then case (iii) represents the optimum situation. Thus whenever  $m_1$  is present in the mapping function, an optimization procedure (without additional constraints) as presented here leads to the trivial solution of all  $m_n$  except  $m_1$  equal to zero, when  $m_1$  is given by Equation (20). Other non-trivial situations could be investigated by imposing additional constraints such as a priori assignment of numerical values to other  $m_n$ . It should be emphasized, however, that the elliptical optimum shape holds true only in the case of biaxial stress fields.

#### Case iv:

The fourth case ( $m_3 = -.09$ ,  $p = 1$ ,  $q = -1$ ) represents pure shear loading, which does not permit an optimum elliptical shape. However, the present constrained optimization procedure leads  $m_3$  to  $-0.09$ , at which point  $I = 0.9077$ . This optimized hole is also square-like (double barrel shape) with rounded corners. For comparison, the circular hole in this stress field will produce an  $I = 1.0$ . The maximum stress concentration is 3.07 at  $\beta = 23.6^\circ$  ( $\beta = 0$  corresponds to one of the applied stress directions) which compares favorably with a stress concentration of 4 for a circular hole in a large plate under pure shear.

#### EXAMPLE II

The cases discussed so far dealt with the mapping function containing only one term. Inclusion of an additional term in the mapping function leads to considerable complexity in the algebraic manipulations. The mapping function considered was

$$z = \zeta + \frac{m_3}{\zeta^3} + \frac{m_7}{\zeta^7} \quad (21)$$

In this case, the symbolic expression for  $I$ , the stress integral, and its

derivatives were obtained; however, these were so unwieldy that a direct numerical approach was followed. It was found that, in the case of a uniaxial stress field ( $p = 1, q = 0$ ), the minimum value of  $I = 0.3623$  occurred at  $m_3 = -0.05$  and  $m_7 = 0.0035$ . It is interesting to note that this case implies an improvement over the shape obtained by case (ii) of the last example. The actual difference between the two cases is, however, very small.

### CONCLUSIONS

A stress integral has been developed which can be effectively used to optimize a number of hole shapes in structures consisting of large plates. For example, it has been possible to obtain a stress concentration factor of 2.47 for a square-like (double barrel shape) hole with rounded corners. This value is about 3% lower than the optimized quasi square shape and about 5% lower than the double barrel shape described by Durelli and Rajaiah.<sup>1</sup> Both shapes occurred in uniaxially loaded plates.

In the case of pure shear loading of large plates, another square-like (double barrel shape) hole shape has been found which reduces the maximum stress concentration to 3.07 from a value of 4 for a circular hole.

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APPENDIX - CONSTANTS FOR EQUATIONS (4) AND (5)

$$a_7 = -m_7 S$$

$$\begin{aligned} a_5 = & ((15 m_5 m_7^3 + (-9 m_3 - 5)m_5 - 6 m_1 m_3 + 6 m_1)m_7^2 + (3 m_1 m_5^2 \\ & + (-6 m_3 + m_1^2 + 3)m_5 - 2 m_1)m_7 + 3 m_5^3 + m_1 m_5^2 + (m_3 - 1)m_5)S + 3 D m_7^2 \\ & - D m_7)/(15 m_7^3 + (9 m_3 - 5)m_7^2 + (-3 m_1 m_5 - m_1^2 - 3)m_7 - 3 m_5^2 \\ & - m_1 m_5 - m_3 + 1) \end{aligned}$$

$$\begin{aligned} a_3 = & ((15 m_3 m_7^3 + (-10 m_1 m_5 + 9 m_3^2 + 5 m_3)m_7^2 + (-10 m_5^2 - 3 m_1 m_3 m_5 \\ & + 6 m_3^2 + (m_1^2 - 3)m_3 - 2 m_1^2)m_7 - 3 m_3 m_5^2 + (m_1 m_3 - 2 m_1)m_5 + m_3^2 \\ & - m_3)S - D m_1 m_7 - D m_5)/(15 m_7^3 + (9 m_3 - 5)m_7^2 + (-3 m_1 m_5 - m_1^2 \\ & - 3)m_7 - 3 m_5^2 - m_1 m_5 - m_3 + 1) \end{aligned}$$

$$\begin{aligned} a_1 = & ((15 m_1 m_7^3 + (30 m_5 + 9 m_1 m_3 - 5 m_1)m_7^2 + ((-3 m_1^2 - 10)m_5 \\ & - 6 m_1 m_3 - m_1^3 + 3 m_1)m_7 - 3 m_1 m_5^2 + (-6 m_3 - m_1^2)m_5 - m_1 m_3 \\ & - m_1)S + 3 D m_7 - D)/(15 m_7^3 + (9 m_3 - 5)m_7^2 + (-3 m_1 m_5 - m_1^2 - 3)m_7 \\ & - 3 m_5^2 - m_1 m_5 - m_3 + 1) \end{aligned}$$

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**2. DEPARTMENTAL REPORTS, A SEMIFORMAL SERIES, CONTAIN INFORMATION OF A PRELIMINARY, TEMPORARY, OR PROPRIETARY NATURE OR OF LIMITED INTEREST OR SIGNIFICANCE. THEY CARRY A DEPARTMENTAL ALPHANUMERICAL IDENTIFICATION.**

**3. TECHNICAL MEMORANDA, AN INFORMAL SERIES, CONTAIN TECHNICAL DOCUMENTATION OF LIMITED USE AND INTEREST. THEY ARE PRIMARILY WORKING PAPERS INTENDED FOR INTERNAL USE. THEY CARRY AN IDENTIFYING NUMBER WHICH INDICATES THEIR TYPE AND THE NUMERICAL CODE OF THE ORIGINATING DEPARTMENT. ANY DISTRIBUTION OUTSIDE DTNSRDC MUST BE APPROVED BY THE HEAD OF THE ORIGINATING DEPARTMENT ON A CASE-BY-CASE BASIS.**